

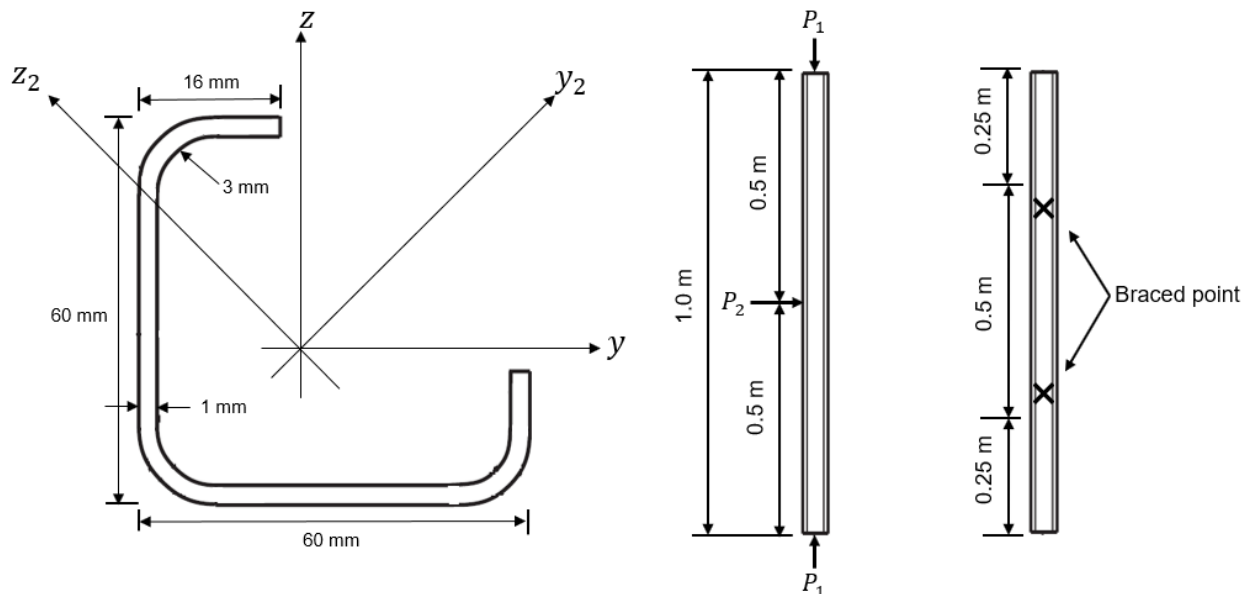
## EC3 1-3 2006 CFFD Example 007

### ANGLE-SECTION MEMBER WITH LIPS UNDER COMBINED COMPRESSION, BENDING, AND SHEAR

#### EXAMPLE DESCRIPTION

Compression, moment, and shear capacities and demand/capacity ratio are calculated for Angle section with lips at mid-height as shown below. It is simply supported with a length of 1.0 meter. The member is point-braced against buckling geometric minor axis and lateral-torsional buckling at 250 mm from each end.

#### GEOMETRY, PROPERTIES AND LOADING



Loads:  $P_1 = 2200 \text{ N}$ ,  $P_{2,y} = 500 \text{ N}$ ,  $P_{2,z} = 300 \text{ N}$

#### TECHNICAL FEATURES TESTED

- Axial compressive strength
- Major moment strength
- Shear strength
- Demand/Capacity ratio.

## COMPUTER FILE: EC3 1-3 2006 CFFD Ex007

### Applicable Programs

➤ SAP2000

### RESULTS COMPARISON

Independent results are hand calculated.

### CONCLUSION

The results show exact match with independent results.

### Benchmarks: SAP2000

Output Parameter	Program	Independent	Percent Difference
Axial - Flexural buckling $N_{b,Rd} (N)$	23106	23103	0.01%
Axial – Torsional-Flexural buckling $N_{b,Rd} (N)$	15805	15810	0.03%
Axial – Local & Distortional Buckling $N_{c,Rd} (N)$	30740	30741	0.00%
Flexure – Lateral-Torsional Buckling $M_{b,Rd} (N - mm)$	584449	584296	0.03%
Flexure – Local & Distortional Buckling $M_{c,Rd} (N - mm)$	732122	732200	0.01%
Shear $V_{b,Rd} (N)$	11893	11899	0.05%
D/C Ratio	0.703	0.703	0.00%

## HAND CALCULATION

### Properties:

Material:  $E = 210,000 \text{ N/mm}^2$ ,  $G = 80,770 \text{ N/mm}^2$ ,  $f_{yb} = 350 \text{ N/mm}^2$

Section:  $h = b = 60 \text{ mm}$ ,  $t = 1 \text{ mm}$ ,  $c = 16 \text{ mm}$ ,  $r = 3 \text{ mm}$

$$\rightarrow h_p = b_p = b - t = 60 - 1 = 59 \text{ mm}$$

$$\rightarrow c_p = c - t/2 = 16 - 1/2 = 15.5 \text{ mm}$$

Check for the effect of rounding of the corners:

$$\frac{r}{t} = \frac{3}{1} = 3 < 5 \rightarrow OK$$

$$\frac{r}{b_p} = \frac{3}{59} = 0.051 < 0.1 \rightarrow OK$$

Therefore, the effect of rounded corners can be neglected in calculation of section properties:

$$A_g = 149 \text{ (mm}^2\text{)}$$

$$I_{y2} = 109481.708 \text{ (mm}^4\text{)}$$

$$I_{z2} = 34457.802 \text{ (mm}^4\text{)}$$

$$i_{y2} = 27.107 \text{ (mm)}$$

$$i_{z2} = 15.207 \text{ (mm)}$$

$$W_{el} = 2624.259 \text{ (mm}^3\text{)}$$

$$I_t = 49.667 \text{ (mm}^4\text{)}$$

$$I_w = 6468558.7 \text{ (mm}^6\text{)}$$

$$y_0 = 30.792 \text{ (mm)}$$

$$z_0 = 0.0 \text{ (mm)}$$

Member: for the section at mid-height within the middle segment of the member

$$K_y = K_z = K_T = 1.0 \text{ for a pinned-pinned condition}$$

$$L_y = 1000 \text{ mm}, L_z = L_T = 500 \text{ mm}$$

$$k_{yy} = k_{zz} = k_{zy} = k_{yz} = 1.0$$

Loadings:  $P_1 = 2200 \text{ N}$ ,  $P_{2,y} = 500 \text{ N}$ ,  $P_{2,z} = 300 \text{ N}$

Required strengths: for the section in the middle

$$N_{Ed} = P_1 = 2200 \text{ (N)}$$

$$M_{Ed,y} = \frac{P_{2,y}L}{4} = \frac{500 \times 1000}{4} = 125000 \text{ (N - mm)}$$

$$M_{Ed,z} = \frac{P_{2,z}L}{4} = \frac{300 \times 1000}{4} = 75000 \text{ (N - mm)}$$

$$M_{Ed,y2} = M_{Ed,y}\cos 45^\circ + M_{Ed,z}\sin 45^\circ = 125000\cos 45^\circ + 75000\sin 45^\circ = 141421.356 \text{ (N - mm)}$$

$$M_{Ed,z2} = -M_{Ed,y}\sin 45^\circ + M_{Ed,z}\cos 45^\circ = -125000\sin 45^\circ + 75000\cos 45^\circ = -35355.339 \text{ (N - mm)}$$

$$V_{Ed,y} = \frac{P_{2,y}}{2} = \frac{500}{2} = 250 \text{ (N)}$$

**Member Compression Capacity:** the compression capacity is calculated considering the limit states of global buckling, and local and distortional buckling.

1. Local and distortional buckling:

The effective width method is utilized to calculate the nominal axial strength in consideration of local and distortional buckling with the compressive stress of  $f_{yb} = 350 \text{ (N/mm}^2\text{)}$ .

Check for the applicability of the method as the following conditions are satisfied:

$$\begin{aligned}\frac{b}{t} &= \frac{60}{1} = 60 \rightarrow OK \\ \frac{c}{t} &= \frac{16}{1} = 16 < 50 \rightarrow OK \\ \frac{c}{b} &= \frac{16}{60} = 0.27 \rightarrow 0.2 < \frac{c}{b} < 0.6 \rightarrow OK\end{aligned}$$

As the section is subjected to uniform compression and both flanges have identical dimension, they are considered partially stiffened elements with a simple lip edge stiffener and have the same effective properties. The calculation below is only shown for one flange:

$$\begin{aligned}\psi &= 1 \\ k_\sigma &= 4 \\ \varepsilon &= \sqrt{\frac{235}{f_{yb} [N/mm^2]}} = \sqrt{\frac{235}{350}} = 0.8194 \\ \bar{\lambda}_{p,b} &= \frac{b_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{59/1}{28.4 \times 0.8194\sqrt{4}} = 1.268 > 0.673 \\ \rho &= \frac{\bar{\lambda}_{p,b} - 0.055(3 + \psi)}{\bar{\lambda}_{p,b}^2} = \frac{1.268 - 0.055(3 + 1)}{1.268^2} = 0.652 \leq 1.0 \\ b_{eff} &= \rho b_p = 0.652 \times 59 = 38.465 \text{ (mm)} \\ b_{e1} &= b_{e2} = 0.5b_{eff} = 0.5 \times 38.465 = 19.233 \text{ (mm)}\end{aligned}$$

The lip is considered an outstand (unstiffened) element under uniform compression:

$$\begin{aligned}\frac{c_p}{b_p} &= \frac{15.5}{59} = 0.27 < 0.35 \rightarrow k = 0.5 \\ \bar{\lambda}_{p,c} &= \frac{c_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{15.5/1}{28.4 \times 0.8194\sqrt{0.5}} = 0.942 > 0.748 \\ \rho &= \frac{\bar{\lambda}_{p,c} - 0.188}{\bar{\lambda}_{p,b}^2} = \frac{0.942 - 0.188}{0.942^2} = 0.85 \leq 1.0 \\ c_{eff} &= \rho c_p = 0.85 \times 15.5 = 13.17 \text{ (mm)}\end{aligned}$$

The stiffener consisting of  $b_{e2}$  of the flange and  $c_{eff}$  of the lip (Figure 1) is subjected to distortional buckling ( $b_{e1}$  of the flange is not affected by distortional buckling and not included in the iterative procedure below):

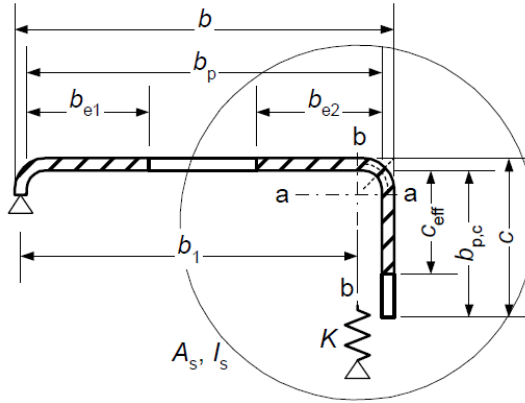


Figure 1 Edge Stiffener (Dubina et al., 2012)

**1<sup>st</sup> iteration:**

$$b_1 = b_2 = b_p - \frac{tb_{e2}^2}{t(b_{e2} + c_{eff})} = 59 - \frac{1 \times \frac{19.233^2}{2}}{1(19.233 + 13.17)} = 53.29 \text{ (mm)}$$

$$A_{s1} = A_{s2} = t(b_{e2} + c_{eff}) = 1(19.233 + 13.17) = 32.403 \text{ (mm}^2\text{)}$$

$$k_{f1} = \frac{A_{s2}}{A_{s1}} = 1$$

$$K = \frac{Et^3}{4(1 - \nu^2)} \frac{1}{(b_1^2 h_p + b_1^3 + 0.5b_1 b_2 h_p k_f)}$$

$$= \frac{210,000 \times 1^3}{4(1 - 0.3^2)} \frac{1}{(53.29^2 \times 59 + 53.29^3 + 0.5 \times 53.29 \times 53.29 \times 59 \times 1)} = 0.143 \text{ (N/mm}^2\text{)}$$

$$I_s = \frac{t(t^2 b_{e2}^2 + 4b_{e2} c_{eff}^3 + t^2 b_{e2} c_{eff} + c_{eff}^4)}{12(b_{e2} + c_{eff})}$$

$$= \frac{1(1^2 \times 19.233^2 + 4 \times 19.233 \times 13.17^3 + 1^2 \times 19.233 \times 13.17 + 13.17^4)}{12(19.233 + 13.17)} = 531 \text{ (mm}^4\text{)}$$

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s} = \frac{2\sqrt{0.143 \times 210,000 \times 531}}{32.403} = 246.7 \text{ (N/mm}^2\text{)}$$

$$\bar{\lambda}_d = \sqrt{f_{yb}/\sigma_{cr,s}} = \sqrt{350/246.7} = 1.19 \rightarrow 0.65 < \bar{\lambda}_d < 1.38$$

$$\chi_d = 1.47 - 0.723\bar{\lambda}_d = 1.47 - 0.723 \times 1.2 = 0.609$$

Since  $\chi_d = 0.609 < 1.0 \rightarrow$  iteration is required.

**2<sup>nd</sup> iteration:**

$b_{e2}$  of the flange and  $c_{eff}$  of the lip are subjected to reduced stress  $\sigma_{com,Ed} = \chi_d f_{yb} / \gamma_{M0}$  such that:

$$\bar{\lambda}_{p,b,red} = \bar{\lambda}_{p,b} \sqrt{\chi_d} = 1.268 \times \sqrt{0.609} = 0.989 > 0.673$$

$$\rho = \frac{\bar{\lambda}_{p,b,red} - 0.055(3 + \psi)}{\bar{\lambda}_{p,b,red}^2} = \frac{0.989 - 0.055(3 + 1)}{0.989^2} = 0.786 \leq 1.0$$

$$b_{e2} = 0.5b_{eff} = 0.5\rho b_p = 0.5 \times 0.786 \times 59 = 23.19 \text{ (mm)}$$

$$\bar{\lambda}_{p,c,red} = \bar{\lambda}_{p,c} \sqrt{\chi_d} = 0.942 \times \sqrt{0.609} = 0.735 < 0.748 \rightarrow \rho = 1.0$$

$$c_{eff} = \rho c_p = 1.0 \times 15.5 = 15.5 \text{ (mm)}$$

$$b_1 = b_2 = b_p - \frac{\frac{tb_{e2}^2}{2}}{t(b_{e2} + c_{eff})} = 59 - \frac{1 \times \frac{23.19^2}{2}}{1(23.19 + 15.5)} = 52.05 \text{ (mm)}$$

$$A_{s1} = A_{s2} = t(b_{e2} + c_{eff}) = 1(23.19 + 15.5) = 38.69 \text{ (mm}^2\text{)}$$

$$k_{f1} = \frac{A_{s2}}{A_{s1}} = 1$$

$$K = \frac{Et^3}{4(1 - \nu^2)} \frac{1}{(b_1^2 h_p + b_1^3 + 0.5b_1 b_2 h_p k_f)}$$

$$= \frac{210,000 \times 1^3}{4(1 - 0.3^2)} \frac{1}{(52.05^2 \times 59 + 52.05^3 + 0.5 \times 52.05 \times 52.05 \times 59 \times 1)} = 0.1515 \text{ (N/mm}^2\text{)}$$

$$I_s = \frac{t(t^2 b_{e2}^2 + 4b_{e2} c_{eff}^3 + t^2 b_{e2} c_{eff} + c_{eff}^4)}{12(b_{e2} + c_{eff})}$$

$$= \frac{1(1^2 \times 23.19^2 + 4 \times 23.19 \times 15.5^3 + 1^2 \times 23.19 \times 15.5 + 15.5^4)}{12(23.19 + 15.5)} = 870.26 \text{ (mm}^4\text{)}$$

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s} = \frac{2\sqrt{0.1515 \times 210,000 \times 870.26}}{38.69} = 272 \text{ (N/mm}^2\text{)}$$

$$\bar{\lambda}_d = \sqrt{f_{yb}/\sigma_{cr,s}} = \sqrt{350/272} = 1.134 \rightarrow 0.65 < \bar{\lambda}_d < 1.38$$

$$\chi_d = 1.47 - 0.723\bar{\lambda}_d = 1.47 - 0.723 \times 1.134 = 0.65$$

Since  $\chi_d = 0.65 \neq 0.609$  from previous iteration, more iterations are carried out and the final iteration gives:

$$\chi_d = 0.65.$$

$$b_{e2} = 22.664 \text{ (mm)}$$

$$c_{eff} = 15.37 \text{ (mm)}$$

$$b_{e1} = 19.233 \text{ (mm)}$$

$$A_{eff} = 2tb_{e1} + 2\chi_d t(b_{e2} + c_{eff})$$

$$= 2 \times 1 \times 19.233 + 2 \times 0.65 \times 1(22.664 + 15.37) = 87.83 \text{ (mm}^2\text{)}$$

$$A_{eff} = 87.83 \text{ (mm}^2\text{)} < 149 \text{ (mm}^2\text{)} = A_g$$

$$\rightarrow N_{c,Rd} = \frac{A_{eff} f_{yb}}{\gamma_{M0}} = \frac{87.83 \times 350}{1.0} = 30740.5 \text{ (N)}$$

Because the section is symmetric about  $y_2$  axis, its effective properties are also symmetric about  $y_2$  axis, resulting in  $e_{Ny} = 0 \rightarrow \Delta M_{y,Ed} = 0$

$$\begin{aligned}\bar{z} &= \frac{\sum_i A_i z_i}{A} = \frac{2tb_p \frac{b_p}{2} \cos 45^\circ + 2tc_p \left(b_p + \frac{c_p}{2}\right) \cos 45^\circ}{A} \\ &= \frac{59 \times 59 \cos 45^\circ + 2 \times 15.5 \times \left(59 + \frac{15.5}{2}\right) \cos 45^\circ}{149} = 26.34 \text{ (mm)} \\ \bar{z}_{eff} &= \frac{\sum_i A_{eff,i} z_i}{A_{eff}} = \frac{2tb_{e1} \frac{b_{e1}}{2} \cos 45^\circ + 2\chi_d tb_{e2} \left(b_p - \frac{b_{e2}}{2}\right) \cos 45^\circ + 2\chi_d tc_{eff} \left(b_p + \frac{c_{eff}}{2}\right) \cos 45^\circ}{A_{eff}} \\ &= \frac{\left[19.233 \times 19.233 + 2 \times 0.65 \times 22.664 \left(59 - \frac{22.664}{2}\right) + 2 \times 0.65 \times 15.37 \times \left(59 + \frac{15.37}{2}\right)\right] \cos 45^\circ}{87.83} \\ &= 25 \text{ (mm)} \\ e_{Nz} &= \bar{z}_{eff} - \bar{z} = 26.34 - 25 = 1.34 \text{ (mm)} \\ \Delta M_{z,Ed} &= N_{Ed} e_{Nz} = 2200 \times 1.34 = 2950 \text{ (N-mm)}\end{aligned}$$

2. Global buckling: includes flexural buckling and torsional and flexural-torsional buckling

- i. Flexural buckling: although the member is braced against buckling about minor geometric axis and torsional buckling at the locations of 250 mm and 750 mm from the bottom end, the flexural buckling is considered about principal axes and the unbraced length for both principal major and minor axes of buckling is taken as the longer unbraced length of buckling about geometric axes, which is:

$$L_{y2} = L_{z2} = \max(L_y, L_z) = \max(1000 \text{ mm}, 500 \text{ mm}) = 1000 \text{ mm}$$

$$N_{cr,y2} = \frac{\pi^2 EI_{y2}}{(K_y L_{y2})^2} = \frac{\pi^2 (210,000) 109481.708}{(1.0 \times 1000)^2} = 226913.64 \text{ (N)}$$

$$N_{cr,z2} = \frac{\pi^2 EI_{z2}}{(K_y L_{z2})^2} = \frac{\pi^2 (210,000) 34457.802}{(1.0 \times 1000)^2} = 71417.82 \text{ (N)}$$

$$\bar{\lambda}_{y2} = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr,y2}}} = \sqrt{\frac{87.83 \times 350}{226913.64}} = 0.368$$

$$\bar{\lambda}_{z2} = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr,z2}}} = \sqrt{\frac{87.83 \times 350}{71417.82}} = 0.656$$

For Angle section with lips, the buckling curve is  $c$  and  $\alpha = 0.49$

$$\Phi_{y2} = 0.5[1 + \alpha(\bar{\lambda}_{y2} - 0.2) + \bar{\lambda}_{y2}^2] = 0.5[1 + 0.49(0.368 - 0.2) + 0.368^2] = 0.609$$

$$\Phi_{z2} = 0.5[1 + \alpha(\bar{\lambda}_{z2} - 0.2) + \bar{\lambda}_{z2}^2] = 0.5[1 + 0.49(0.656 - 0.2) + 0.656^2] = 0.827$$

$$\chi_{y2} = \frac{1}{\Phi_{y2} + \sqrt{\Phi_{y2}^2 - \bar{\lambda}_{y2}^2}} = \frac{1}{0.609 + \sqrt{0.609^2 - 0.368^2}} = 0.914$$

$$\chi_{z2} = \frac{1}{\Phi_{z2} + \sqrt{\Phi_{z2}^2 - \bar{\lambda}_{z2}^2}} = \frac{1}{0.827 + \sqrt{0.827^2 - 0.656^2}} = 0.752$$

$$N_{by2,Rd} = \frac{\chi_{y2} A_{eff} f_{yb}}{\gamma_{M1}} = \frac{0.914 \times 87.83 \times 350}{1.0} = 28096.7 \text{ (N)}$$

$$N_{bz2,Rd} = \frac{\chi_{z2} A_{eff} f_{yb}}{\gamma_{M1}} = \frac{0.752 \times 87.83 \times 350}{1.0} = 23103.1 \text{ (N)}$$

ii. Torsional and flexural-torsional buckling:

$$i_0 = \sqrt{i_{y2}^2 + i_{z2}^2 + y_0^2 + z_0^2} = \sqrt{27.107^2 + 15.207^2 + 30.792^2 + 0^2} = 43.751 \text{ (mm)}$$

$$N_{cr,T} = \frac{1}{i_0^2} \left[ G I_t + \frac{\pi^2 E I_w}{L_T^2} \right] = \frac{1}{43.751^2} \left[ 80,770 \times 49.667 + \frac{\pi^2 210,000 \times 6468558.7}{(1.0 \times 500)^2} \right] = 30112 \text{ (N)}$$

$$\beta = 1 - \frac{y_0^2 + z_0^2}{i_0^2} = 1 - \frac{30.792^2 + 0^2}{43.751^2} = 0.505$$

$$N_{cr,TF} = \frac{N_{cr,y2}}{2\beta} \left[ 1 + \frac{N_{cr,T}}{N_{cr,y2}} - \sqrt{\left(1 - \frac{N_{cr,T}}{N_{cr,y2}}\right)^2 + 4 \left(\frac{y_0}{i_0}\right)^2 \frac{N_{cr,T}}{N_{cr,y2}}} \right]$$

$$= \frac{226913.64}{2 \times 0.505} \left[ 1 + \frac{30112}{226913.64} - \sqrt{\left(1 - \frac{30112}{226913.64}\right)^2 + 4 \left(\frac{30.792}{43.751}\right)^2 \frac{30112}{226913.64}} \right] = 28120 \text{ (N)}$$

As  $N_{cr,TF} = 28120 \text{ (N)} < 30112 \text{ (N)} = N_{cr,T}$

$$\rightarrow \bar{\lambda}_T = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr,TF}}} = \sqrt{\frac{87.83 \times 350}{28120}} = 1.045$$

For Angle section with lips, the buckling curve is  $c$  and  $\alpha = 0.49$

$$\Phi_T = \frac{0.5[1 + \alpha(\bar{\lambda}_T - 0.2) + \bar{\lambda}_T^2]}{1} = \frac{0.5[1 + 0.49(1.045 - 0.2) + 1.045^2]}{1} = 1.253$$

$$\chi_T = \frac{1}{\Phi_T + \sqrt{\Phi_T^2 - \bar{\lambda}_T^2}} = \frac{1}{1.253 + \sqrt{1.253^2 - 1.045^2}} = 0.514$$

$$N_{b,Rd} = \frac{\chi_T A_{eff} f_{yb}}{\gamma_{M1}} = \frac{0.514 \times 87.83 \times 350}{1.0} = 15810 \text{ (N)}$$

**Member Flexural Capacity:** the flexural capacity is calculated considering the limit states of lateral-torsional buckling, and local and distortional buckling.

## 1. Local and distortional buckling:

The effective width method is utilized to calculate the nominal flexural strength in consideration of local and distortional buckling with the compressive stress in the top flange of  $f_{yb} = 350 \text{ (N/mm}^2\text{)}$ . As the section is subjected to positive moment, the top flange is under stress gradient and it is considered a partially stiffened element with a simple lip edge stiffener:

$$\sigma_1 = f_{yb} = 350 \text{ (N/mm}^2\text{)}$$

$\sigma_2 = 0.0 \text{ (N/mm}^2\text{)}$  as this end of the flange is at the neutral axis in the middle of the section for bending about principal major axis

$$\psi = \frac{\sigma_2}{\sigma_1} = \frac{0.0}{350} = 0.0$$

$$k_\sigma = \frac{8.2}{1.05 + \psi} = \frac{8.2}{1.05 + 0.0} = 7.81$$

$$\bar{\lambda}_{p,b} = \frac{b_p/t}{28.4\epsilon\sqrt{k_\sigma}} = \frac{59/1}{28.4 \times 0.8194\sqrt{7.81}} = 0.907 > 0.673$$

$$\rho = \frac{\bar{\lambda}_{p,b} - 0.055(3 + \psi)}{\bar{\lambda}_{p,b}^2} = \frac{0.907 - 0.055(3 + 0)}{0.907^2} = 0.902 \leq 1.0$$

$$b_{eff} = \rho b_p = 0.902 \times 59 = 53.22 \text{ (mm)}$$

$$b_{e1} = 0.4b_{eff} = 0.4 \times 53.22 = 21.28 \text{ (mm)}$$

$$b_{e2} = 0.6b_{eff} = 0.6 \times 53.22 = 31.93 \text{ (mm)}$$

The bottom flange is in tension and calculation of its effective width is not needed.

The lip is considered an outstand (unstiffened) element under uniform compression:

$$\frac{c_p}{b_p} = \frac{15.5}{59} = 0.337 < 0.35 \rightarrow k = 0.5$$

$$\bar{\lambda}_{p,c} = \frac{c_p/t}{28.4\epsilon\sqrt{k_\sigma}} = \frac{15.5/1}{28.4 \times 0.8194\sqrt{0.5}} = 0.942 > 0.748$$

$$\rho = \frac{\bar{\lambda}_{p,c} - 0.188}{\bar{\lambda}_{p,c}^2} = \frac{0.942 - 0.188}{0.942^2} = 0.85 \leq 1.0$$

$$c_{eff} = \rho c_p = 0.85 \times 15.5 = 13.17 \text{ (mm)}$$

The stiffener consisting of  $b_{e1}$  (instead of  $b_{e2}$  because  $b_{e1}$  is adjacent to the lip) of the flange and  $c_{eff}$  of the lip is subjected to distortional buckling ( $b_{e2}$  of the flange is not affected by distortional buckling and not included in the iterative procedure below):

**1<sup>st</sup> iteration:**

$$b_1 = b_p - \frac{tb_{e1}^2}{2} = 59 - \frac{1 \times \frac{21.28^2}{2}}{1(21.28 + 13.17)} = 52.43 \text{ (mm)}$$

$$b_2 = 0$$

$$A_{s1} = t(b_{e1} + c_{eff}) = 1(21.28 + 13.17) = 34.45 \text{ (mm}^2\text{)}$$

$$A_{s2} = 0.0 \text{ (mm}^2\text{)} \text{ as the bottom stiffener is in tension}$$

$$k_f = \frac{A_{s2}}{A_{s1}} = 0.0$$

$$K = \frac{1}{4(1 - \nu^2) (b_1^2 h_p + b_1^3 + 0.5b_1 b_2 h_p k_{f1})}$$

$$= \frac{210,000 \times 1^3}{4(1 - 0.3^2)} \frac{1}{(52.43^2 \times 59 + 52.43^3 + 0.5 \times 52.43 \times 52.43 \times 59 \times 0.0)} = 0.188 \text{ (N/mm}^2\text{)}$$

$$I_s = \frac{t(t^2 b_{e1}^2 + 4b_{e1}c_{eff}^3 + t^2 b_{e1}c_{eff} + c_{eff}^4)}{12(b_{e1} + c_{eff})}$$

$$= \frac{1(1^2 \times 21.28^2 + 4 \times 21.28 \times 13.17^3 + 1^2 \times 21.28 \times 13.17 + 13.17^4)}{12(21.28 + 13.17)} = 545 \text{ (mm}^4\text{)}$$

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s} = \frac{2\sqrt{0.188 \times 210,000 \times 545}}{34.45} = 269.5 \text{ (N/mm}^2\text{)}$$

$$\bar{\lambda}_d = \sqrt{f_{yb}/\sigma_{cr,s}} = \sqrt{350/269.5} = 1.14 \rightarrow 0.65 < \bar{\lambda}_d < 1.38$$

$$\chi_d = 1.47 - 0.723\bar{\lambda}_d = 1.47 - 0.723 \times 1.14 = 0.646$$

Since  $\chi_d = 0.646 < 1.0 \rightarrow$  iteration is required.

## 2<sup>nd</sup> iteration:

$b_{e1}$  of the flange and  $c_{eff}$  of the lip are subjected to reduced stress  $\sigma_{com,Ed} = \chi_d f_{yb}/\gamma_{M0}$ :

$$\bar{\lambda}_{p,b,red} = \bar{\lambda}_{p,b}\sqrt{\chi_d} = 0.907 \times \sqrt{0.646} = 0.729$$

$$\rho = \frac{\bar{\lambda}_{p,b,red} - 0.055(3 + \psi)}{\bar{\lambda}_{p,b,red}^2} = \frac{0.729 - 0.055(3 + 0)}{0.729^2} = 1.06 > 1.0 \rightarrow \rho = 1.0$$

$$b_{eff} = \rho b_p = 1.0 \times 59 = 59 \text{ (mm)}$$

$$b_{e1} = 0.4b_{eff} = 0.4 \times 59 = 23.6 \text{ (mm)}$$

$$\bar{\lambda}_{p,c,red} = \bar{\lambda}_{p,c}\sqrt{\chi_d} = 0.942 \times \sqrt{0.646} = 0.757 > 0.748$$

$$\rho = \frac{\bar{\lambda}_{p,c,red} - 0.188}{\bar{\lambda}_{p,c,red}^2} = \frac{0.757 - 0.188}{0.757^2} = 0.993 \leq 1.0$$

$$c_{eff} = \rho c_p = 0.993 \times 15.5 = 15.4 \text{ (mm)}$$

$$b_1 = b_p - \frac{tb_{e1}^2}{t(b_{e1} + c_{eff})} = 59 - \frac{1 \times 23.6^2}{1(23.6 + 15.4)} = 51.86 \text{ (mm)}$$

$$A_{s1} = t(b_{e1} + c_{eff}) = 1(23.6 + 15.4) = 39 \text{ (mm)}$$

$$A_{s2} = 0.0 \text{ (mm}^2\text{)} \text{ as the bottom stiffener is in tension}$$

$$k_{f1} = \frac{A_{s2}}{A_{s1}} = 0.0$$

$$K = \frac{Et^3}{4(1 - \nu^2)} \frac{1}{(b_1^2 h_p + b_1^3 + 0.5b_1 b_2 h_p k_{f1})}$$

$$= \frac{210,000 \times 1^3}{4(1 - 0.3^2)} \frac{1}{(51.86^2 \times 59 + 51.86^3 + 0.5 \times 51.86 \times 51.86 \times 59 \times 0.0)} = 0.1935 \text{ (N/mm}^2\text{)}$$

$$I_s = \frac{t(t^2 b_{e1}^2 + 4b_{e1}c_{eff}^3 + t^2 b_{e1}c_{eff} + c_{eff}^4)}{12(b_{e1} + c_{eff})}$$

$$= \frac{1(1^2 \times 23.6 + 4 \times 23.6 \times 15.4^3 + 1^2 \times 23.6 \times 15.4 + 15.4^4)}{12(23.6 + 15.4)} = 857 \text{ (mm}^4\text{)}$$

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s} = \frac{2\sqrt{0.1935 \times 210,000 \times 857}}{39} = 303 \text{ (N/mm}^2\text{)}$$

$$\bar{\lambda}_d = \sqrt{f_{yb}/\sigma_{cr,s}} = \sqrt{350/303} = 1.075 \rightarrow 0.65 < \bar{\lambda}_d < 1.38$$

$$\chi_d = 1.47 - 0.723\bar{\lambda}_d = 1.47 - 0.723 \times 1.075 = 0.693$$

Since  $\chi_d = 0.693 \neq 0.646$  from previous iteration, more iterations are carried out and the final iteration gives:

$$\chi_d = 0.685 \rightarrow t_{eff} = \chi_d t = 0.685 \times 1.0 = 0.685 \text{ (mm)}$$

$$b_{e1} = 23.6 \text{ (mm)}$$

$$c_{eff} = 15.08 \text{ (mm)}$$

$$b_{e2} = 31.93 \text{ (mm)}$$

The neutral axis of the section with effective top flange and lip measured from the top of the section is:

$$\begin{aligned} \bar{y} &= \frac{\sum_i A_i y_i}{A} = \frac{\left[ tb_{e2} \left( b_p - \frac{b_{e2}}{2} \right) + \chi_d t b_{e1} \frac{b_{e1}}{2} + \chi_d t c_{eff} \times \frac{c_{eff}}{2} + t b_p \left( b_p + \frac{b_p}{2} \right) + t c_p \left( 2b_p - \frac{c_p}{2} \right) \right] \cos 45^\circ}{tb_{e2} + \chi_d t b_{e1} + \chi_d t c_{eff} + t b_p + t c_p} \\ &= \frac{\left[ 31.93 \left( 59 - \frac{31.93}{2} \right) + 0.685 \times 23.6 \frac{23.6}{2} + 0.685 \times 15.08 \frac{15.08}{2} + 59 \left( 59 + \frac{59}{2} \right) + 15.5 \left( 2 \times 59 - \frac{15.5}{2} \right) \right] \cos 45^\circ}{31.93 + 0.685 \times 23.6 + 0.685 \times 15.08 + 59 + 15.5} \\ &= \frac{6062}{132.926} = 45.6 \text{ (mm)} \end{aligned}$$

$$W_{eff,c} = 2092 \text{ (mm}^3\text{)}$$

$$W_{eff,t} = 2465 \text{ (mm}^3\text{)}$$

$$M_{c,Rd} = \frac{W_{eff,c} f_{yb}}{\gamma_{M0}} = \frac{2092 \times 350}{1.0} = 732200 \text{ (N - mm)}$$

Similar calculation is repeated to determine the effective section modulus about the minor axis. Under the moment about principal minor axis, both lips are in compression simultaneously and both flanges are in stress gradient. The effective elastic section modulus is calculated to be

$$W_{eff,z,c} = 969.7 \text{ (mm}^4\text{)}$$

$$W_{eff,z,t} = 1167 \text{ (mm}^4\text{)}$$

$$M_{cz,Rd} = M_{bz,Rd} = \frac{W_{eff,z,c} f_{yb}}{\gamma_{M0}} = \frac{969.7 \times 350}{1.0} = 339395 \text{ (N - mm)}$$

## 2. Lateral-torsional buckling:

Due to the concentrated loading and simply support condition at both ends of the column:

$$C_1 = 1.365, C_2 = 0.553, C_3 = 1.73$$

$$k_w = 1.0 \text{ and } K_{LTB} = 1.0$$

$z_a = 40.375 \text{ (mm)}$  as the load is applied on the top flange along the geometric z-z axis

$$z_g = z_a - z_s = 40.375 - 0 = 40.375 \text{ (mm)}$$

$$z_j = 0.0 \text{ (mm)} \text{ along } z_2 \text{ axis as the section is symmetric about } y_2 \text{ axis}$$

$$L_{cr} = 500 \text{ (mm)}$$

$$I_t = 49.667 \text{ (mm}^4\text{)}$$

$$I_{z2} = 34457.802 \text{ (mm}^4\text{)}$$

$$I_w = 6468558.7 \text{ (mm}^6\text{)}$$

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{L_{cr}^2} \left\{ \left[ \left( \frac{K_{LTB}}{k_w} \right) \frac{I_w}{I_z} + \frac{L_{cr}^2 G I_T}{\pi^2 E I_z} + (C_2 z_g - C_3 z_j)^2 \right]^{0.5} - (C_2 z_g - C_3 z_j) \right\}$$

$$= 1612562 \text{ (N - mm)}$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{eff,y} f_{yb}}{M_{cr}}} = \sqrt{\frac{2092 \times 350}{1612562}} = 0.674$$

The applicable buckling curve is  $b$  and  $\alpha_{LT} = 0.34$

$$\Phi_{LT} = 0.5 \left[ 1 + \alpha_{LT} (\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2 \right] = 0.5 \left[ 1 + 0.34 (0.674 - 0.2) + 0.674^2 \right] = 0.808$$

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} = \frac{1}{0.808 + \sqrt{0.808^2 - 0.674^2}} = 0.798 \leq 1.0$$

$$M_{b,Rd} = \chi_{LT} W_{eff,y} \frac{f_{yb}}{\gamma_{M1}} = 0.798 \times 2092 \frac{350}{1.0} = 584296 \text{ (N - mm)}$$

## Member Shear Capacity:

Shear capacity is calculated along the geometric z-z axis:

$$\bar{\lambda}_w = 0.346 \frac{s_w}{t} \sqrt{\frac{f_{yb}}{E}} = 0.346 \frac{59}{1} \sqrt{\frac{350}{210000}} = 0.833 \rightarrow 0.83 < \bar{\lambda}_w < 1.40$$

$$f_{bv} = \frac{0.48 f_{yb}}{\bar{\lambda}_w} = \frac{0.48 \times 350}{0.833} = 201.6 \text{ (N/mm}^2\text{)}$$

$$V_{b,Rd} = \frac{h_w t f_{bv}}{\gamma_{M0}} = \frac{59 \times 1 \times 201.6}{1.0} = 11899 \text{ (N)}$$

## Combined D/C ratio:

The ratio by Equation 6.36 in Eurocode 3 1-3 2006 would provide the largest D/C ratio and govern the design:

$$\frac{D}{C} = \left( \frac{N_{Ed}}{N_{b,Rd}} \right)^{0.8} + \left( \frac{M_{y,Ed} + \Delta M_{y,Ed}}{M_{by,Rd}} \right)^{0.8} + \left( \frac{M_{z,Ed} + \Delta M_{z,Ed}}{M_{bz,Rd}} \right)^{0.8}$$

$$= \left( \frac{2200}{15810} \right)^{0.8} + \left( \frac{141421.356 + 0}{584296} \right)^{0.8} + \left( \frac{35355.339 + 2950}{339395} \right)^{0.8} = 0.703$$